Looking for Victims Theory

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Speech Outline

- Basics on identification
- Alternative composite hypotheses
- Inference : Configuration
- Inference : Priors
- Inference : Likelihood
- Inference : Posteriors

Basics on Identification

- Let’s start from the “standard” paternity identification case.
- Evidence=Mother,Child and Alleged father DNA
- Usually results are given in terms of a likelihood ratio

$$LR = \frac{\Pr(E \mid H = 1)}{\Pr(E \mid H = 2)}$$

- Identification consists on the computation of the Evidence probability assuming:
- The AF paternity (H=1)

Versus

- Someone else is the father of the Child (H=2)

Basics on identification

- H=1 has a precise meaning, not necessarily H=2
- Often, neither the mother nor the AF are interested in better specifying H=2
- Very often H=2 is assumed to be a “random man”
- Very very often belonging to the same ethnic group as the AF
- NOTE: H=1,2 are **not** random variables in a LR, they are just two mutually excluding conjectures
- RESULT: The probability to observe the evidence if AF is the father is $x \in [0, \infty]$ times the evidence probability, if H=2 holds true

Basics on identification

- PROBLEM: Can I use the LR approach if more alternatives to H=1 exist, i.e. if an alternative composite hypothesis arises?
- NO: at most it is possible to evaluate a LR for each of the alternative hypothesis' components. Each ratio assumes the alternative component as the only existing one :

  many ratios = high confusion
- If we want to compare the main hypothesis with all the possible alternatives they must be considered all together

Basics on Identification

- Solution: introduce prior probabilities on the each of the alternative component hypothesis: the overall set of hypotheses becomes a random variable

Basics on Identification

Two main results

1) We can compute via Bayes theorem the posterior probability for each conjecture taking into account the others:

$$\Pr(H=1 \mid E) = \frac{\Pr(E \mid H=1)\Pr(H=1)}{\Pr(E \mid H=1)\Pr(H=1) + \Pr(E \mid H=2a)\Pr(H=2a) + \ldots + \Pr(E \mid H=2c)\Pr(H=2c)}$$

2) We can compute the Weight of Evidence (WE), i.e. the Bayesian version of the LR

$$\frac{\Pr(H=1 \mid E)}{\Pr(H=2a \cup \ldots \cup 2c \mid E)} = \frac{\Pr(E \mid H=1)}{\Pr(E \mid H=2a \cup \ldots \cup 2c)} \frac{\Pr(H=1)}{\Pr(H=2a \cup \ldots \cup 2c)}$$

Alternative composite hypotheses in paternity identification

Consider:

- just one diallelic locus: A=(A,a)
- Evidence: Mother=“Aa”; Child=“aa”; Alleged Father=“aa”

Population 1:

- Pr(a)=0.1; Pr(A)=0.9; Pr(aa)=0.01; Pr(aA)=0.18; Pr(AA)=0.81

Population 2

- Pr(a)=0.9; P(A)=0.1; Pr(aa)=0.81; Pr(aA)=0.18; Pr(AA)=0.01

Hypotheses:

- H=1: “Alleged father” is the true father
- H=2-a A generic member of Population 1 is the true father
- H=2-b A generic member of Population 2 is the true father
- H=2-c The brother of alleged father is the true father
- ….

Alternative composite hypotheses

	H=1:AF	H=2a:Pop1
Pr(H)	0.5	0.5
Pr(H|E)	0.91	0.09

$$LR = \frac{\Pr(E \mid H = 1)}{\Pr(E \mid H = 2a)} = 10{,}11$$

Alternative composite hypotheses

	H=1:AF	H=2a: Pop1	H=2b: Pop2
Pr(H)	0.5	0.25	0.25
Pr(H|E)	0.666	0.033	0.30

$$WE = \frac{\Pr(E \mid H = 1)}{\Pr(E \mid H = 2a \cup 2b)} = \frac{0.666}{0.333} = 2$$

$$LR_a = \frac{\Pr(E \mid H = 1)}{\Pr(E \mid H = 2a)} = \frac{0.9091}{0.09} = 10.11$$

$$LR_b = \frac{\Pr(E \mid H = 1)}{\Pr(E \mid H = 2b)} = \frac{0.5263}{0.4737} = 1.11$$

WE= Weight of Evidence (i.e. the Bayesian version of the LR, essentially an “average” using prior probabilities as weights)

Alternative composite hypotheses

	H=1:AF	H=2a: Pop1	H=2b:Pop2	H=2c: Brother
Pr(H)	0.5	0.1666	0.1666	0.1666
Pr(H|E)	0.659	0.022	0.1978	0.1208

$$WE = \frac{\Pr(E \mid H = 1)}{\Pr(E \mid H = 2a \cup 2b \cup 2c)} = 1.93$$

$$LR_a = \frac{\Pr(E \mid H = 1)}{\Pr(E \mid H = 2a)} = 10.11$$

$$LR_b = \frac{\Pr(E \mid H = 1)}{\Pr(E \mid H = 2b)} = 1.11$$

$$LR_c = \frac{\Pr(E \mid H = 1)}{\Pr(E \mid H = 2c)} = 1,818$$

Alternative composite hypotheses : conclusions

- The Bayesian approach (WE) is attractive since it provides support to the main hypothesis taking into account all the alternatives at the same time
- Since it gives more, it demands more: prior probabilities on the hypotheses are required
- Identification of the victims of a mass disaster: we need to take into account many alternatives for each main hypothesis. Fortunately we can use some convincing non-informative prior probabilities

Why are many alternatives relevant in the Mass Disaster Identification problem?

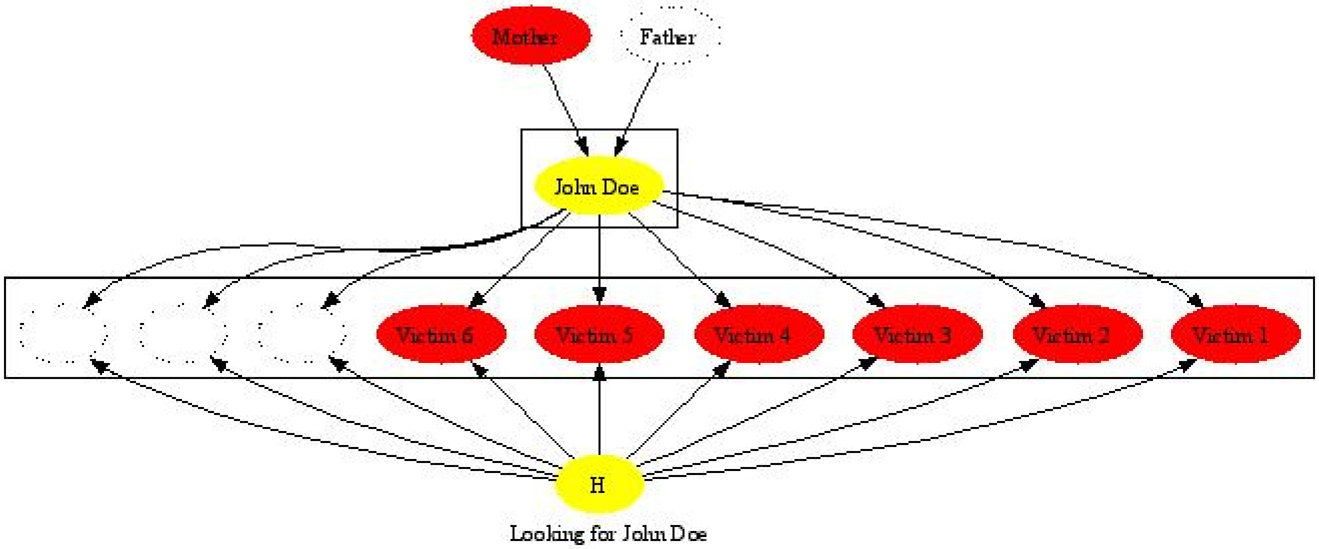
- Assume a disaster has occurred
- N persons were involved
- n(V) victims were recovered
- n(M) missing persons were named
- A question of possible interest arises…

A question of interest

- The family of John Doe is interested in finding JD among the victims, so they ask:
- "Is victim 12 John Doe?"
- What is behind the alternative "John Doe is not Victim 12"? There are some specific alternatives:

1) JD is one of the other n(V)-1 recovered bodies
2) JD has still not been recovered

A graph representing the problem



A question of interest

- To derive the posterior identification probability for JD with respect to all the recovered victims (including Victim 12), we need a prior probability on each of them. Since we assume to have only DNA evidence, a non-informative prior probability is a sensible choice

$$\Pr(H_{JW} = 1) = \cdots \Pr(H_{JW} = n(V)) = \frac{1}{N}$$

$$\Pr(H_{JW} = \mathrm{Re}\, st) = \frac{N - n(V)}{N}$$

A question of interest

- After a non-informative prior probability is given to the hypothesis, the posterior probabilities, conditionally on the evidence, can be provided

H_{JW}	V1	….	V12	V13	Rest
$Pr(H_{JD})$	0.01	0.01	0.01	0.01	0.87
$Pr(H_{JD\|}\|E)$	0.1	…..	0.88	0.02	0.01

Note: to evaluate if JD is Victim 12, we used information on all the other victims obtaining a probability for each of the components of the hypothesis

A question of interest

- If required, it is possible to derive the WE from prior and posterior hypothesis probabilities

$$
\begin{aligned}
WE &= \frac{\Pr(E \mid H = 12)}{\Pr(E \mid H = 1 \cup \cdots \cup H = 13 \cup H = rest)} \\
&= \frac{\Pr(H = 12 \mid E)}{1 - \Pr(H = 12 \mid E)} \frac{1 - \Pr(H = 12)}{\Pr(H = 12)}
\end{aligned}
$$

Another question of interest

- Also the family of Mario Rossi is interested in finding MR among the victims
- Is it correct to proceed performing the same analysis, i.e. starting from a non-informative prior probability and deriving the posterior identification probability for all the victims????
- A possible result of such an approach

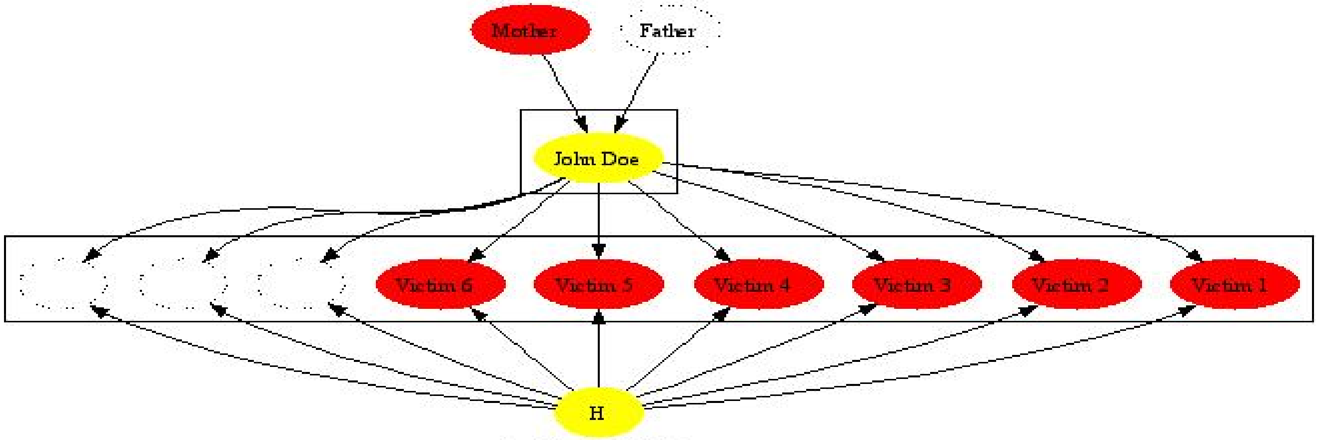
Another question of interest

H	V1	….	V12	V13	Rest
$Pr(H_{JD})$	0.01	…..	0.01	0.01	0.87
$Pr(H_{JD}\|E)$	0.11	…..	0.82	0.02	0.01
$Pr(H_{MR})$	0.01	…..	0.01	0.01	0.87
$Pr(H_{MR}\|E)$	0.94	…..	0.01	0.02	0.01

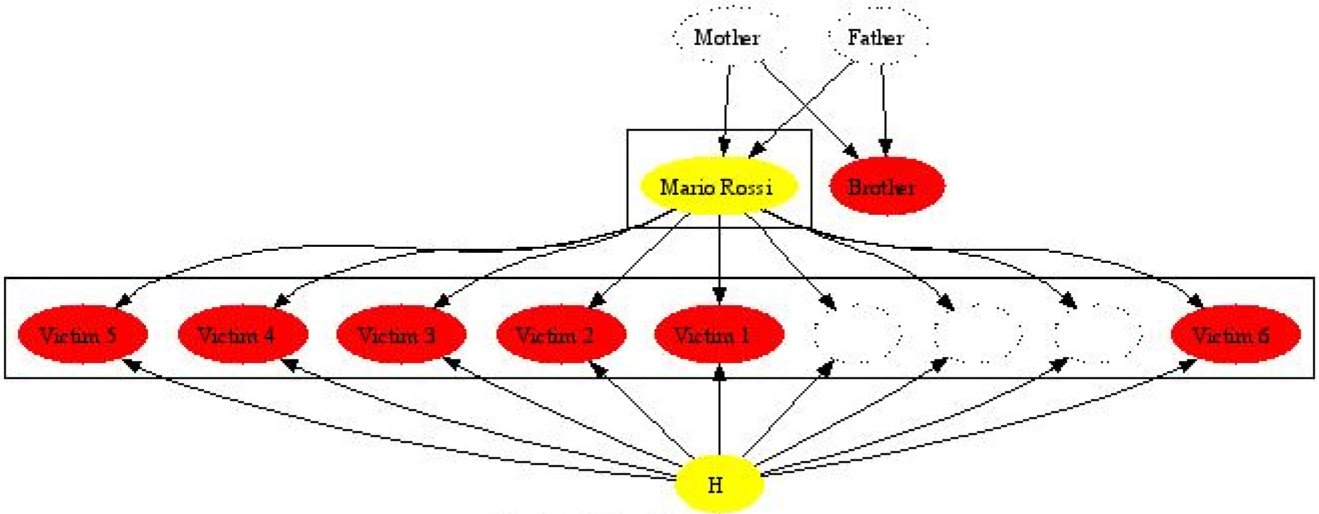
Another question of interest

- Note: victim “1” has the probability of being one of the two missing persons with a probability >1!!!
- This incoherency is because the hypothesis random variables are actually dependent, since they consider the same victims with respect to different missing individuals

AQI: two separate analyses

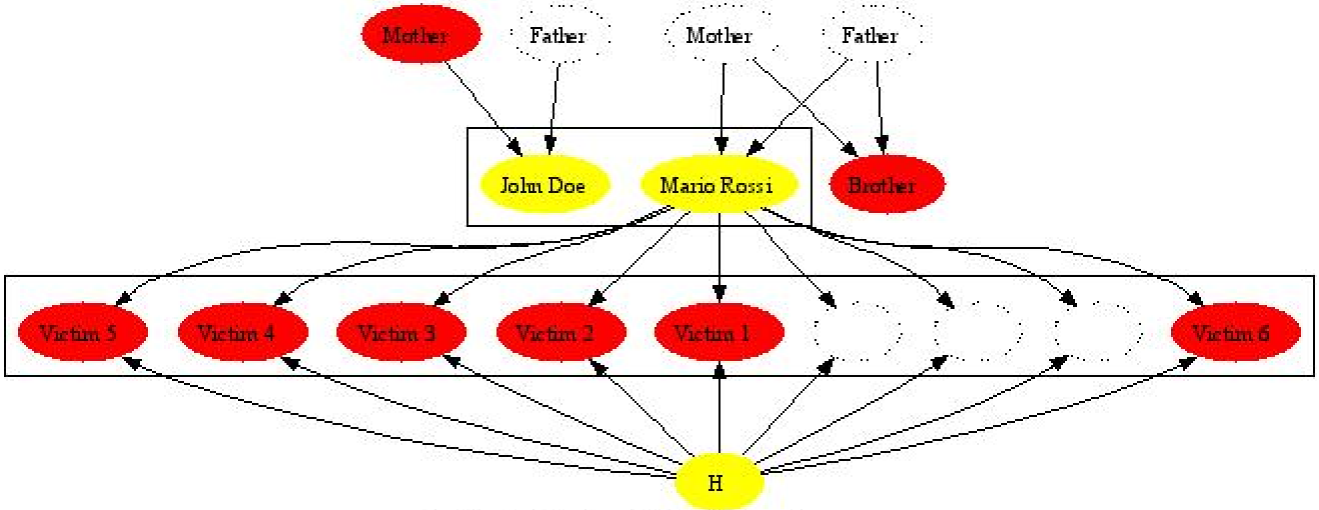


Looking for John Doe



Looking for Mario Rossi

AQI: One integrated analysis



Looking for John Doe AND Mario Rossi

Other questions of interest

- Obviously, other questions of interest concern all the missing persons included in the search
- Note: In this way the dual problem is also considered, i.e. the probability that a corpse belongs to one of the missing individuals or to someone else still nameless. The possible question could be:
- “What is the probability that victim 7 is one of the missing persons searched for by their families or is among those still unnamed??”

Inference: the configuration hypothesis random variable

- What about the hypothesis random variable when several victims and missing persons are considered all together?

The Missing H	1	2	3	4	n(M)	…	…	…	N
1	V1	V7	…	…	V5	…	V16	…	V76
2	…	…	…	…	…	V5	V8	V7	V32
….	V2	V4	V8	V44	V77	…	…	…	…

Inference: the main steps (1)

- Recognize that the number of possible identifiable victims are in this range:

$$\left[\left(\max(0, n(M) + n(V), \min(n(M) + N(V)\right)\right]$$

- Produce all the possible states of the configuration hypothesis NOT assigning the same victim to more than one missing person
- Assume a non-informative prior probability for each of these states

Inference: the main steps (2)

- Provide posteriors on the configuration hypothesis random variable via:

❖ A propagation in a Probabilistic Expert System (PES)

❖ Algebraic solution as provided in Cavallini-Corradi (2008)

Inference: the main steps (3)

- Derive marginal probabilities for each victims and each missing persons summing up all over the posterior configuration hypothesis probabilities.

Missings H	1	2	3	4	n(M)	…	…	N	Pr(H|E)
1	V2	…	…	V7	V12	V71	V88	…	0.15
2	V1	V7	…	…	V5	…	V16	…	0.09
….	…	…	…	…	…	V5	V8	V7	0.01
N(H)	V2	V4	V8	V44	V77	…	…	…	0.06

Conclusions

L4V provides:

- for all the missing persons the probabilities to be one of the recovered victims or to be still not recovered
- for all missing persons the probabilities to be one of the recovered victims or to be still not recovered