Some mathematical problems in the DNA identification of victims in the 2004 tsunami and similar mass fatalities

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Received 27 July 2005; received in revised form 16 October 2005; accepted 1 November 2005
Available online 19 December 2005

Abstract

DNA is a major and essential identification tool for mass fatality incidents including the hundreds of thousands of victims of the 2004 Indian Ocean tsunami. Mathematical complications characteristic of this sort of mass fatality include prevalence of related victims, the many races represented among the victims, and various identification modalities in tandem with DNA. Four mathematical problems of interest are discussed in this paper. (1) Other quantifiable factors (i.e. geography) can be formally accounted for by including a likelihood ratio that can be thought of as reducing the “effective number of victims.” (2) When a victim is found and tentatively identified as V, but then it comes to light that the victim has a relative W who is also missing, confidence in the identity is depressed. To account for the existence of W, increment the effective number of victims by the likelihood ratio supporting W as the identity of the victim. (3) When several apparently related victims are found, their mutual identities should be calculated simultaneously. Compared to one-at-a-time, serial identifications, this is both logical and may lead to much more confidence in the identities. (4) Although there may be many different population groups represented among the missing, it is generally sufficient to consider population statistics for only a few of them in deciding whether to declare an identification.

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Keywords: DNA identification; Mass fatality; Kinship; Tsunami identification

1. Introduction

DNA is unique among identification modalities in lending itself well to mathematical treatment. The purpose of this paper is to give some mathematical guidance and suggestions for determining identities of victims in the case of mass disasters similar to the December 2004 tsunami deaths in the Indian Ocean. The general outline of strategy for DNA-based mass fatality identification is reasonably clear from experience of this and earlier incidents [1–5]. DNA profiles of victim DNA (“PM” for post-mortem) are compared with reference profiles that come from relatives (indirect references) and/or from relics such as a toothbrush of the victim (direct reference). A likelihood ratio comparing appropriate hypotheses may then reasonably be interpreted using a Bayesian model [6] – in other words, assuming some prior odds – to give posterior odds of correct identification (1). A typical policy is that identification will be declared provided the posterior odds are high enough that the chance of a misidentification is very small (2).

On the other hand, the discipline of mass identification is not yet mature. Every disaster, particularly a gigantic one, has unique aspects, and it is already apparent what some of the novel mathematical complications of tsunami identifications are. The emphasis in this paper is on DNA calculations and a Bayesian approach. Beyond that the only unifying theme is relevance to the practical problem.
2. Methods

2.1. Population allele frequencies

Population allele frequencies are from 15 Identifiler loci in Thai [7], Chinese [8], Caucasian [9], African-American [9], Japanese [10], Indian [8], Vietnamese [11], Korean [11], African [12,13].

2.2. LR computation

Kinship LR computations are performed using the Symbolic Kinship [14] module of the author’s DNA-VIEW software. The module also includes a facility to create simulated identification examples (http://dna-view.com/simulate.htm), used for several of the examples herein. Simulations used Thai allele frequencies [7], used for several of the examples herein.

2.3. Racial distances

For present purposes “race” simply means a population that has been isolated – i.e. endogamous – for a period of time and whose allele frequencies have therefore drifted compared to other populations. Allele frequency differences between races provide a possible way to assign individuals to race [15–17]. They also represent a possible pitfall when deciding on the confidence of a putative identification, because calculating with the wrong reference population creates a bias toward false confidence in the certainty of an identification. I therefore define “racial distance” as the typical ratio by which a DNA profile randomly selected from race r is expected to be more probable in race r than in race s. This is a useful statistic for evaluating the suitability of using a few reference races as proxies.

Racial distance estimates are calculated [15–17] from available population samples [7–13] as follows. Let the indices r, l, and a range over races, loci, and alleles within a locus. Consider the probability q(r, l, a) of finding allele a at locus l in race r. Assuming the existence of allele population data for r, a collection of numbers c(r, l, a), the numbers of times the allele was counted in a population sample, a plausible estimate (especially for length polymorphisms) of q is the sample frequency

\[ q(r, l, a) = \frac{c(r, l, a)}{\sum_a c(r, l, a)} \]

Also, define “extended-sample” frequencies \( q' \) based on extending the sample by any one allele

\[ q'(r, l, a) = \frac{1 + c(r, l, a)}{1 + \sum_a c(r, l, a)} \]

If a profile \( E \) is simulated from population r by Monte Carlo selection from \( c(r, l, a) \) then \( L_E(r, s) = \Pi_{a \in E}[q(r, l, a)/q'(s, l, a)] \) is the likelihood ratio favoring r over population s as the origin of \( E \). (Extended-sample frequencies in the denominator compensate for acertainment bias from using the numerator population data to choose a profile [15].) The “typical” (that is, expected in the sense of geometric mean) racial discrimination likelihood ratio from a full profile with two alleles per locus is then [15]

\[ D(r, s) = \Pi_{a \in E} \left[ \frac{q(r, l, a)}{q'(s, l, a)} \right]^{2q(r, l, a)} \]

\( D(r, s) \) can be calculated for any pair of races for which allele population data exists, and is a directed distance measure between the two races that is natural in a forensic context because it measures the expected error in using the wrong reference population for computing a random match probability.

By the Central Limit Theorem the distribution of \( \log L_E \) is nearly normal if the number of loci is large. Comparison of moments with Monte Carlo samples of 100,000 profiles confirms that the deviations from normality, with 15 locus profiles, are small. Kurtosis and skew are 3.3 and \(-1/6\), versus 3 and 0 for a normal distribution.

3. Discussion

3.1. Framework

A simplified example illustrates the Bayesian framework that I assume herein. Suppose \( v + 1 \) people are lost including
a person Victor. A body V turns up. Without saying anything more the probability is \( 1/(v+1) \) – i.e. the prior odds are \( 1:v \) meaning \( 1/v \) – that the body is Victor. Various evidence, such as DNA and the size and location of the body, combine to give a likelihood ratio of \( L \) supporting the proposition that the body is Victor rather than anyone else. Bayes’ Theorem, formulated in terms of odds, says that

\[
\text{posterior odds} = (\text{prior odds}) \times (\text{likelihood ratio}) \quad (1)
\]

so the posterior odds are \( L/v \) that \( V \) is Victor. Since probability = odds/(odds + 1), the posterior probability is very nearly \( 1 - v/L \) so long as (as will be true in situations of interest) \( L \) is much larger than \( v \). Each declared identification has some (small) probability \( v/L \) to be a mistake; \( v/L \) can also be interpreted as an expected (in the statistical sense) number of misidentifications in declaring that \( V = \) Victor.

Perhaps we have in mind a probabilistic budget – no doubt less than \( 1 \) – for the total number of expected errors to be accumulated over all declared identifications. Apportioning that budget among individual identifications suggests, as a baseline policy for declaring each identification, to insist on a stringent standard that

\[
\text{expected number of misidentifications per identification} = \frac{v}{L} < \varepsilon \quad (2)
\]

for some small \( \varepsilon \) such as \( \varepsilon = 1/10,000 \). The simplest policy is to keep the same \( \varepsilon \) for all victims, but it may be judicious to make exceptions occasionally while keeping an eye on the bottom line.

If \( L \) comes from several independent pieces of evidence, such as location found (geography), physical attributes, and DNA, then it can be expressed as a product of the likelihood ratios for each different piece of evidence:

\[
L = (\text{geographical LR}) \times (\text{physical LR}) \times (\text{DNA LR}) \quad (3)
\]

Substituting (3) into (1) and multiplying left to right (instead of multiplying all the likelihood ratio factors together first as in (3)) corresponds to the idea of climbing a ladder of evidence wherein each rung represents the posterior odds relative to the evidence below it, and at the same time is prior odds relative to the next evidence. That is

\[
\text{posterior-to-geography odds} = (\text{prior odds}) \times (\text{geographical LR}) \quad \text{and}
\]

\[
\text{prior-to-physical odds} = \text{posterior-to-geography odds};
\]

\[
\text{posterior-to-geography-and-physical odds} = (\text{prior-to-physical odds}) \times (\text{physical LR}) \text{ etc.} \quad (4)
\]

**Effective number of victims**

Suppose \( v + 1 = 10,000 \) are missing in Thailand, 500 of them are missing from some particular seaside village SV, and assume that all 500 of these plus 10% of the 400 missing from a nearby village to the south will wash up near SV. If Kanya is missing from SV, what are the odds that a random body \( V \) is Kanya?

Geographical LR = \( \frac{X}{V} \)

where \( X = \text{Pr}(V \text{ appears near SV}|V = \text{Kanya}) = 1; \ Y = \text{Pr}(V \text{ appears near SV}|V \neq \text{Kanya}) = (499/v) + (400 \times 10%/v) = 539/v \) since there are \( v \) victims in Thailand who are not Kanya, and 499 victims from SV who are not Kanya. So \( LR = v/539 \) and

\[
\text{posterior-to-geography odds} = \left( \frac{1}{v} \right) \frac{v}{539} = \frac{1}{539}
\]

a rather obvious and expected result. It is convenient to refer to 540 – or 539, why split hairs? – as the **effective number of victims** after consideration of geography. This number is relative to the facts that the victim of interest comes from SV and that the body was found at SV; otherwise it would be a different number.

Since the tsunami hit people in their homes and on vacations, it is certain that many related victims are involved. Family members perishing together is a typical complication of airplane crashes [3,18] – it was a conspicuous feature in sorting out some of the identities from the 1998 Swissair 111 crash [4] near Halifax and from the post-9/11 crash in Queens of AA587. Among the World Trade Center victims there were not many who were related, and we did not systematically consider the problems of related victims—in one instance early on only by luck was a mistaken identification between brothers avoided.

Without related victims, the alternative identity hypotheses to consider are merely

\( \mathbf{H}_p \). This PM profile represents missing person \( X \),

\( \mathbf{H}_o \). This PM profile is unrelated to \( X \).

When relatives \( X_1, X_2, \ldots \) are also possible victims, additional hypotheses must be calculated separately

\( \mathbf{H}_i \). This PM profile is \( X \)'s relative \( X_i \).

The tsunami disaster scale, nature, and uncertainties require approaching related victim problems systematically. There are several different complications to consider.

3.2. *One of related victims found*

The tsunami differs from airplane crashes in that many victims will never be recovered at all, which brings an extra complication when there is a tentative identification for a
missing victim who has relatives that are also, and still, missing. Suppose that brothers Ray and Sam are missing, and only Ray’s daughter J is available as a reference (Fig. 1). A body, V, is found, and comparing the DNA of V and of J suggests that they might be father and daughter (H0) – that V might be Ray. But of course the possibility H1 that V is Sam must also be considered; otherwise the probability Pr(H0) that V is Ray would be vastly overstated as a result of comparing it only with the “strawman” H0.

In a typical situation there are prior odds Odds(Ray) that the body is Ray and let us assume Odds(Ray) = Odds(Sam) = 1/v, v + 1 being the effective number of victims. Let L(Ray, 0) be the likelihood ratio by which the DNA evidence favors Ray as the identity of V rather than an unrelated identity. Then if there were no Sam to consider, the posterior odds that V is Ray would be

\[
\text{Odds}(V = \text{Ray}|\text{DNA, no Sam}) = \frac{L(\text{Ray}, 0)}{v}
\]  

(5)

In view of the existence of Sam, however, the posterior odds that V = Ray are different, and, from Eq. (14) (Appendix) are expressed in a simple way as a combination of L(Ray, 0) and L(Sam, 0):

\[
\text{Odds}(V = \text{Ray}|\text{DNA}) \approx \frac{L(\text{Ray}, 0)}{L(\text{Sam}, 0) + v}
\]  

(6)

Comparing (6) and (5) reveals the elegant rule that the possibility that the body is Sam rather than Ray is accounted for by imagining L(Sam, 0) additional effective victims.

For example, L(Ray, 0) = 600,000, L(Sam, 0) = 5400, and v = 600. Then if Sam were neglected the odds, from (5), that V = Ray would be 1000:1, but taking Sam into account per (6) this 10-fold smaller, 100:1. The expected number of misidentifications, per (2), is correspondingly 10 times larger.

3.3. Multiple related victims found

When several bodies are found that are suspected of being members of the same family (Table 1) and are to be identified (Fig. 1): Is the body Ray or is it Sam?

Table 1

<table>
<thead>
<tr>
<th>Hypothesis Hi, i=</th>
<th>vw</th>
<th>v</th>
<th>w</th>
<th>s</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>V = Sue,</td>
<td>3,867,000</td>
<td>96.1</td>
<td>125.4</td>
<td>7406</td>
<td>1</td>
</tr>
<tr>
<td>W = Joe</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V = Sue,</td>
<td>96.1</td>
<td>125.4</td>
<td>7406</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>W unrelated</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V unrelated,</td>
<td></td>
<td>125.4</td>
<td>7406</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>W = Joe</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V &amp; W sibs but not of F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V, W, F all unrelated</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Correct analysis – simultaneous consideration of all data to evaluate both identities

\[ p_i = \text{prior likelihood} = 1 \]  
\[ w_i = \text{posterior likelihood} = 3,867,000 \]  
\[ W_i = \text{posterior probability} = 99.5\% \]  
\[ p_i = \text{prior likelihood} = 1/20 \]  
\[ w_i = \text{posterior likelihood} = 3,867,000 \]  
\[ W_i = \text{posterior probability} = 99.95\% \]  

Incorrect analysis – identify victims serially using only presumably known identities at each stage

\[ p_i = \text{prior likelihood} = 19 \]  
\[ w_i = \text{posterior likelihood} = 1826 \]  
\[ W_i = \text{posterior probability} = 83.5\% \]  
\[ p_i = \text{prior likelihood} = 1 \]  
\[ w_i = \text{posterior likelihood} = 3,867,000 \]  
\[ W_i = \text{posterior probability} = 99.95\% \]  

One or both of the two bodies, V and W, may represent missing children Sue and Joe of the typed reference father F. (*) Prior probabilities – (1/20)(1/20), (19/20)(1/20), etc. assuming 20 missing children of each sex – are each multiplied by 400 for convenient comparison. (**) assuming two other missing brother–sister pairs. (†) \( w_i = Li(0) \times p_i \) (‡) \( W_i = w_i/\sum w_i \) (§) prior probabilities of 1/20, 19/20 scaled by factor of 19-20 for ease of comparison. (§) The difference between 361 here and 359 above comes from ignoring H0. (¶) prior probabilities of 1/20, 19/20 scaled by factor of 20 for ease of comparison.
Theorem. The tableau arrangement of Table 1 illustrates the posterior probability of each hypothesis using Bayes’ theorem. We may as well use this approach to simply calculate the likelihood ratios, there is no harm in identifying the likelihood ratios {Li}, and it is convenient to do so.

Example

Table 1 shows a realistic situation where simultaneous consideration of the identities of two victims is important. Two bodies V and W may be the missing daughter and son of the living reference F (hypothesis Hvw; Table 1, vw), and in addition there are at least four other possibilities (Table 1) to consider.

The general method of attack for these problems is to compute and compare the likelihoods {Li} of the DNA conditional upon each of the hypothetical pedigrees H_i, i ∈ {vw, v, w, s, 0}.

L_i = Pr(DNA as observed | H_i)

The ratio L(i, k) = L_i/L_k is then the likelihood ratio by which the DNA supports H_i over H_k.

Note that L(i, k) can equally be expressed as L(i, 0)/L(k, 0). Therefore, and since likelihoods are never used except in ratios, there is no harm in identifying the likelihood ratios {L(i, 0)} as the likelihoods {L_i}, and it is convenient to do so.

Theory—simultaneous and serial identifications

The likelihood ratio L supporting Table 1, vw – identification of both V and W – is at least the ratio L_min = min{L_vw/L_i} by which this explanation exceeds any other, i.e. exceeds whatever is in second place. However, if the second-best explanation is inherently implausible then L_min is needlessly conservative. The accurate expression for L would involve some weighted average of the L_i in the denominator, the weights depending on the respective prior probabilities of the various alternatives H_i. Since, therefore, consideration of prior probabilities is unavoidable, we may as well use them to simply calculate the posterior probability of each hypothesis using Bayes’ Theorem. The tableau arrangement of Table 1 illustrates the calculation: Assuming that relative prior probabilities (“likelihoods”) are given, multiply by the corresponding genetic likelihoods L_i to obtain the posterior likelihoods. These in turn are converted to posterior probabilities by scaling them to sum to 100%. Under the given assumptions it is thus seen to be 99.5% that V is Sue and W is Joe. (The identity of either body alone is slightly higher.)

Consider by contrast how the serial approach would work in this case. If we choose to identify V first, then we compare the hypothesis H_v (that V = Sue) with H_0 (V ≠ Sue), as shown in Table 1. The conclusion that V = Sue is a modest 83.5%, much weaker than the actual value of the evidence. Moreover, to proceed to the next step, using the “established” identity of V to identify W, requires an illogical leap from 83.5% to 100%. Assume that leap is made. Then identifying W amounts to comparing the hypotheses Hvw (V = Sue and W = Joe) with H_i (V = Sue, W unrelated). The resulting confidence of 99.95% is of course an exaggeration, mainly because it rests on an illogical leap, partly because the best alternative explanation H_i (V and W siblings unrelated to F) is overlooked by the serial method.

3.4. Appropriate reference population

Those who were killed in Thailand by the tsunami were about equally Caucasians and Thai, with many other racial groups represented in small proportions (Table 2, derived from http://missingpersons.or.th/index.en.html). An obvious question of practical importance: In making probability calculations is it reasonable to ignore completely those populations which are only scantly represented? Practically speaking, some sort of compromise is inevitable; just as is true in the forensic setting it is always impossible to characterize, let alone to obtain, the precisely appropriate reference population sample.

This section discusses the method and the mathematics to justify considering only the two major constituent victim races.

Analysis for direct id

Suppose a direct match between a body V and some personal effect belonging to a victim Victor. The likelihood

<table>
<thead>
<tr>
<th>Victim Proportion (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thai</td>
</tr>
<tr>
<td>US and European</td>
</tr>
<tr>
<td>Chinese</td>
</tr>
<tr>
<td>Japanese</td>
</tr>
<tr>
<td>Arab</td>
</tr>
<tr>
<td>Malay</td>
</tr>
<tr>
<td>African</td>
</tr>
<tr>
<td>Indian</td>
</tr>
<tr>
<td>Korean</td>
</tr>
<tr>
<td>Vietnamese</td>
</tr>
</tbody>
</table>
ratio (known as “matching odds” in this context) between the hypotheses:

**Hₐ**:  V is Victor

**Hᵦ**:  V is unrelated to Victor

is then \( L = 1/\Pr(M) \), where M is defined as the event that the DNA profile of a randomly selected unrelated body would match that of Victor. The criterion (2) for declaring an identification takes the form

\[
\Pr(M) < \rho, \tag{7}
\]

where the threshold probability \( \rho = \rho(v) = \frac{v}{v} \) and the corresponding contribution to the expected number of misidentifications is \( \Pr(M)v \).

The probability to see any particular DNA profile at random varies from population to population. Therefore, \( \Pr(M) \) is a weighted average

\[
\Pr(M) = \sum_r w_r P_r(M), \quad \text{where} \quad \sum_r w_r = 1 \tag{8}
\]

of the probabilities \( P_r = \Pr(M|\text{race } r) \) from each constituent victim population. The weights \( w_r \) may be the proportions with which each constituent race is represented among the victims – e.g. if subscripts T and C correspond to Thai and Caucasian, \( w_T \) \( \approx w_C \approx 1/2 \) – but in principle \( w_r \) may also vary depending on the circumstances and body appearance in a particular case, i.e. \( w_r \) is the prior likelihood, based on any information other than DNA, that the body is of race \( r \). Therefore, it would not be appropriate simply to use average allele frequencies such as might be compiled from the victim profiles.

For the World Trade Center identifications [5], instead of (7) we adopted the more stringent condition

\[
P_r < \rho, \quad \text{for each } r \tag{9}
\]

Particularly in dealing with tsunami identifications (9) offers two advantages. One, it avoids the need to decide the appropriate weights \( \{w_r\} \). Two, it is likely to be sufficiently conservative to compensate for the sin of not making calculations for some races that are in fact represented. That is, if (9) is satisfied for the main populations, probably (7) is satisfied. If the vast preponderance of the victims are either Thai or Caucasian, it is not far wrong to make calculations just for those two.

An example will illustrate the reason. Suppose \( w_T = 1/100 \) is the prior probability that a given victim is Japanese, and suppose that \( P_T = 10^{-14}, \quad P_C = 10^{-17}, \quad P_3 = 10^{-13} \) – perhaps because \( V \) is Japanese. Then by (8), \( \Pr(M) \approx 10^{-14}/2 + 10^{-17}/2 + 10^{-13}/100 \approx 0.51 \times 10^{-12} \), so \( \Pr(M) < P_T \). Therefore, if (9) holds, so does (7) in this case.

In order to investigate the error in general in ignoring minor races, suppose that (9) holds for races T and C, and suppose for example that \( P_T = \max(P_T, P_C) \). Then the relative error in ignoring minor races is the relative difference, \( \Pr(M)/P_T - 1 \), between \( P_T \) and \( \Pr(M) \). It can be written as a weighted average of the relative errors that would be incurred by substituting \( P_T \) for each of the \( P_r, \quad r = T, C, 3, 4, \ldots \):

\[
\frac{\Pr(M)}{P_T} - 1 = w_T \left( \frac{P_C}{P_T} - 1 \right) + w_3 \left( \frac{P_3}{P_T} - 1 \right)
\]

\[
+ w_4 \left( \frac{P_4}{P_T} - 1 \right) + \cdots \tag{10}
\]

If this expression is non-positive, it is harmless to ignore the minor races for calculation. If it is positive, then it is the relative margin by which \( P_T \) must be less than \( \rho \) in order to achieve the target expected misidentification contribution despite ignoring minor race calculations. The first term on the right is non-positive, and although the proper choice of weights \( w_r \) may be case-specific, at least from the census data \( w_C \) is by far the largest of the coefficients. The next term is positive only if \( P_3 > P_T \) (an obvious prerequisite for ignoring race 3 to be a mistake). To analyze the formula further it is helpful to understand the behavior of \( P_3/P_T \). First, note that it is the likelihood ratio supporting race \( r \) over race \( s \) as the origin of the profile. Therefore, usually \( P_3/P_T > 1 \) when the profile actually comes from population \( r \). Second, I have calculated “typical” (expected in the geometric sense) values of this ratio comparing various populations among various populations of interest (Table 3).

Inter alia these data tend to refute the canard that “Forensic STR markers are very poorly suited to the task

<table>
<thead>
<tr>
<th></th>
<th>Chinese</th>
<th>Thai</th>
<th>Japanese</th>
<th>Indian</th>
<th>Caucasian</th>
<th>African-Am</th>
<th>Mozambican</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinese</td>
<td>1.4 (×3.5)</td>
<td>3.5 (×5.9)</td>
<td>21 (×11)</td>
<td>130 (×27)</td>
<td>750 (×34)</td>
<td>4900 (×67)</td>
<td></td>
</tr>
<tr>
<td>Thai</td>
<td>1.8 (×4.5)</td>
<td>8.5 (×11)</td>
<td>16 (×12)</td>
<td>140 (×32)</td>
<td>920 (×33)</td>
<td>8000 (×72)</td>
<td></td>
</tr>
<tr>
<td>Japanese</td>
<td>2.5 (×5.3)</td>
<td>3.8 (×6.1)</td>
<td>14 (×9)</td>
<td>100 (×22)</td>
<td>790 (×33)</td>
<td>1900 (×45)</td>
<td></td>
</tr>
<tr>
<td>Indian</td>
<td>18 (×16)</td>
<td>8.5 (×11)</td>
<td>23 (×19)</td>
<td>16 (×15)</td>
<td>180 (×27)</td>
<td>480 (×45)</td>
<td></td>
</tr>
<tr>
<td>Caucasian</td>
<td>220 (×32)</td>
<td>98 (×20)</td>
<td>380 (×42)</td>
<td>29 (×13)</td>
<td>48 (×13)</td>
<td>400 (×32)</td>
<td></td>
</tr>
<tr>
<td>African-American</td>
<td>1300 (×63)</td>
<td>1900 (×82)</td>
<td>6900 (×150)</td>
<td>570 (×56)</td>
<td>84 (×39)</td>
<td>1.9 (×6.1)</td>
<td></td>
</tr>
<tr>
<td>Mozambican</td>
<td>9400 (×84)</td>
<td>18000 (×120)</td>
<td>23000 (×170)</td>
<td>2000 (×71)</td>
<td>980 (×67)</td>
<td>3.2 (×5.4)</td>
<td></td>
</tr>
</tbody>
</table>

\( D(r, s) \) supporting the true origin \( r \) (row) of a DNA profile vs. an alternative origin \( s \) (column). Factors in parentheses are the calculated standard deviation. Example: the Japanese/Thai notation 3.8 (×6.1) means that 68% of Japanese profiles would be between 3.8 × 6.1 and 3.8/6.1 times rarer among Thai than among Japanese.
of describing ancestry [19].” DNA forensics has advanced considerably in the 8 years since the research [15] that the quoted statement can be traced back to [20].

Note from Table 3 that the East Asian allele frequencies are very similar between ethnicities (including Korean and Vietnamese, not shown). Therefore, let us group all the East Asian populations other than Thai into a single Vietnamese, not shown). Therefore, let us group all the East Asian populations other than Thai into a single category with \( r = 3 \), and for the moment think of \( r = 4 \) as a mixture of the remaining populations. Eq. (10) is thus truncated at three terms.

If the victim race is 3, East Asian – think Japanese for illustration – then typically \( P_3 \) is only a few (e.g. \( D(3, T) = 3.8 \) times larger than \( P_T \), but many times larger than \( P_C \) (\( D(3, C) = 100 \)). Therefore, we have \( P_C/P_T = D(3, T)/D(3, C) \approx 3.8/100 \), hence \( (P_C/P_T - 1) \) is typically not much different from \(-1\) [and similar considerations show even more strongly that the last term, \( w_A(P_3/P_T - 1) \), is very unlikely to be positive. Therefore, the right hand side of (10) is about \(-w_C + w_3P_3/P_T \) or less, which is safely negative provided that

\[
\frac{P_3}{P_T} < \frac{w_C}{w_3} \tag{11}
\]

Now, the right side of (11) is about 20 to judge from Table 2, and the typical value of \( P_3/P_T \) is \( D(3, T) \) – quite a lot smaller according to Table 3 given the assumption that the victim is East Asian. Therefore (11) probably holds, so not calculating \( P_T \) is unlikely to be a mistake even when it corresponds to the victim’s true race. From simulations, this back-of-envelope estimate is correct about 90% of the time. In 10% of those cases where 3 is the race of origin (10) is positive either because \( P_3/P_T \) is abnormally large or because \( P_C/P_T \) is nearly 1, but both of these circumstances are very unlikely unless \( P_T \) is particularly small [and similar considerations show even more strongly that the last term, \( w_A(P_3/P_T - 1) \), is very unlikely to be positive. Therefore, the right hand side of (10) is about \(-w_C + w_3P_3/P_T \) or less, which is safely negative provided that

\[
\frac{P_3}{P_T} < \frac{w_C}{w_3} \tag{11}
\]

The situation, then, is this: when \( P_T \) is so large that \( (9) \) is barely satisfied, then (10) is negative, which implies (7), i.e. there is no harm in ignoring \( P_3 \). When (10) is positive (about 10% among the times that a simulated victim is actually East Asian), it is because all the probabilities \( P_i \) are so small that \( Pr(M) < \rho \) anyway.

On the infrequent occasions that \( P_3 > P_C > P_T \), again the frequencies are almost certainly all very small.

The same argument easily applies to populations that are not very different from European Caucasian, such as Finns, Arabs, U.S. Hispanic, and Indians. If the prior probability at which the race is represented as a possible identity for the victim is small compared to the “racial distance” (in the sense of Table 3) from a reference race (viz formula 11), then there is little need to bother with a calculation for that population.

The argument may be difficult to apply for those populations that are quite distant from both of the main populations – e.g. African and African-American. Fortunately these seem to be (Table 2) particulary scantily represented among the victims, which suggests that \( w_C/w_A - \) subscript A for African – is safely larger than \( P_A/P_C \) (vide (11)). On the other hand, the body might be in good enough condition to permit judging African skin color or features. Then \( w_C/w_A \) is not large. But in that case \( v \), the effective number of victims taking into account the body appearance, will be much decreased, so (7) will be an easy standard to meet.

Nonetheless, it would seem stubborn to announce the identification of an African victim based on calculating only \( P_T \) and \( P_C \) when African-American [9] and African [12,13] data are readily available. A sensible suggestion is to re-check (9) for the race of identification (or near alternative) before making an announcement.

**Analysis for id by relatives**

If only haplotypes are involved, or partial profiles, then the racial ratios and variations are diminished compared to Table 3, which gives more comfort to the policy of computing only for the major races. Some kinship calculations are in this category. For example, if a body is identified through comparison with a single parent then usually only one allele per locus participates in the likelihood ratio. On the other hand, if both parents are available as references then the ratios and variations are as large as given by Table 3, while unfortunately the likelihood ratio itself is depressed because of large combinatorial factors (e.g. \( 1/8pq \) instead of \( 1/2pq \) for mother–father versus direct id identification).

**4. Summary**

1. To declare an identification, normally the posterior odds supporting the identification should exceed some threshold value decided by policy, probably a policy with the expectation of no misidentifications in the entire mass identification effort. The policy can allow exceptions but with cognizance that exceptions gradually accumulate as accrued chances of error.

2. The “effective number of victims” \( v + 1 \), or equivalently the prior odds \( 1/v \) that a given body is a particular missing person, varies from case to case. The factors to consider include location found compared to expected, and physical attributes observable in the body (including non-DNA expertises such as dental evidence).

3. The confounding effect of related victims is a constant concern. The emphasis in this paper and the greatest concern is when the identification uses living relatives as references rather than direct references but the principle is the same either way. The unknown extra possibilities diminish the confidence of a possible identification. When a direct reference is available for a missing person but there is no reference for a missing relatives of that person, the situation is logically similar to the forensic situation when a suspect tries to explain away his DNA at a crime scene by suggesting that his (untested) brother or other relative is the culprit. Therefore, recommendations 4.2 and 4.4 of the NRC report [21] on forensic DNA evidence would be appropriate.
4. When several possibly related bodies are to be identified using kin references, all reasonable combinations of identities should be enumerated [4] and a calculation made for each combination, so that the identities are evaluated simultaneously.

5. The threshold value for identification should be met by calculations in the principally represented races of Thai and Caucasian, with a third calculation in the race of assignment if it is different. There is no end to possible complications – for example, the reference relatives for a certain missing person might be of various races themselves, implying that the perfect calculation is rather complicated. All models are wrong, but some are useful [22]. There has to be a practical end to the complications we worry about.

Acknowledgements

This paper arose out of discussions on identification strategy during my visit to Phuket, Thailand in March 2005. There were very few DNA profiles available at that time hence the examples in the text are simulated data.

Thanks to P. Sribanditmongkol and David Gross for helpful information about victim demographics. The practical idea to make an extra calculation in the race of attribution is Jon Davoren’s.

Appendix

Suppose one body is found, and from the DNA there are two possible identities to consider, R and S. This situation might arise if R and S are brothers and the identification is through comparison with relatives as discussed in the text.

Let

\[ E \] be the actual DNA type observed in the body

\[ M \] be the event that the body have DNA profile E

\[ H_R \] mean “R is the name of the body”; prior probability \( Pr(H_R) = p_R \)

\[ H_S \] mean “S is the name of the body”; prior probability \( Pr(H_S) = p_S \)

\[ H_0 \] mean neither \( H_R \) nor \( H_S \); prior probability \( Pr(H_0) = 1 - p_R - p_S \); prior probability

\[ Pr(M) \] mean the probability of \( M \) prior to DNA testing.

Now,

\[ Pr(M) = Pr(H_R&M) + Pr(H_S&M) + Pr(H_0&M) \]

\[ = p_R Pr(M|H_R) + p_S Pr(M|H_S) + (1 - p_R - p_S) Pr(M|H_0) \] (12)

Also,

\[ Pr(H_R&M) = Pr(M)Pr(H_R|M) \], hence

\[ Pr(H_R|M) = \frac{p_R Pr(M|H_R)}{Pr(M)} \] (13)

(Bayes’ Theorem) and from (12) and (13):

\[ Pr(\text{not } H_R|M) = \frac{p_S Pr(M|H_S) + (1 - p_R - p_S) Pr(M|H_0)}{Pr(M)} \]

so the posterior odds \( Odds(H_R|M) = Pr(H_R|M)/Pr(\text{not } H_R|M) \) favoring \( H_R \) can be calculated

\[ Odds(H_R|M) = \frac{p_R Pr(M|H_R)}{p_S Pr(M|H_S) + (1 - p_R - p_S) Pr(M|H_0)/L(R, 0)} \]

\[ = (p_S/p_R)(L(S, 0) - 1) + (1 - p_R)/p_R \]

where \( L(x, 0) \) denotes the likelihood ratio \( Pr(M|H_x)/Pr(M|H_0) \) by which the genetic evidence \( M \) favors \( H_x \) over \( H_0 \):

\[ \approx \frac{L(R, 0)}{(p_S/p_R)L(S, 0) + 1/O_R} \] (14)

where \( O_R = p_R/(1 - p_R) \) is the prior odds of \( H_R \).

References


